## Influence of the Gradient of the Weighing Site

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## Summary

The formulas for the dependence between indicated weight and the gradient of the weighing site are deduced and presented in the form of a graph.

This paper consists of page E0...E6.

## 1. Decisive Force Direction for the Weight Indication of Wheel Load Scales

As most commercially available scales a Wheel Load Scale registers the force component rectangular to its active weighing platform.

## 2. Basic Formulas

### 2.1. Formulas for the load shift length and crosswise



$$
\begin{align*}
& \sum y: F-F_{1}-F_{2}=0 \\
& \sum M_{A}: F_{*} * a^{\prime} * \cos \alpha-F_{2^{*}}\left(a^{\prime}+b^{\prime}\right) \star \cos \alpha=0  \tag{1}\\
& \sum M_{B}: F * b^{\prime} * \cos \alpha-F_{1^{*}}\left(a^{\prime}+b^{\prime}\right) * \cos \alpha=0 \tag{2}
\end{align*}
$$

$\mathrm{a}^{\prime}=\mathrm{a}+\mathrm{h} * \tan \alpha$
$\mathrm{b}^{\prime}=\mathrm{b}-\mathrm{h} * \tan \alpha$
$\mathrm{F}_{\mathrm{N} 2}=\mathrm{F}_{2} * \cos \alpha$
$\mathrm{F}_{\mathrm{N} 1}=\mathrm{F}_{1} * \cos \alpha$

Balance of forces
Balance of momentum to point A Balance of momentum to point $B$
(2):
$F_{2}=F * a^{\prime} /\left(a^{\prime}+b^{\prime}\right)$
$F_{1}=F * b^{\prime} /\left(a^{\prime}+b^{\prime}\right)$
(1) $\cap(3) \cap(4)$
$\mathrm{F}_{2}=\mathrm{F} *(\mathrm{a}+\mathrm{h} * \tan \alpha) /(\mathrm{a}+\mathrm{h} * \tan \alpha+\mathrm{b}-\mathrm{h} * \tan \alpha)=\mathrm{F} *(\mathrm{a}+\mathrm{h} * \tan \alpha) /(\mathrm{a}+\mathrm{b})$
(2) $\cap(3) \cap(4)$
$\mathrm{F}_{1}=\mathrm{F} *(\mathrm{~b}-\mathrm{h} * \tan \alpha) /(\mathrm{a}+\mathrm{h} * \tan \alpha+\mathrm{b}-\mathrm{h} * \tan \alpha)=\mathrm{F} *(\mathrm{~b}-\mathrm{h} * \tan \alpha) /(\mathrm{a}+\mathrm{b})$
(6) $\cap(7)$
$\mathrm{F}_{\mathrm{N} 2}=\mathrm{F} * \cos \alpha *(\mathrm{a}+\mathrm{h} * \tan \alpha) /(\mathrm{a}+\mathrm{b})$
(5) $\cap(8)$
$F_{N 1}=F * \cos \alpha *(b-h * \tan \alpha) /(a+b)$
(9) $\cap(10)$
$\mathrm{F}_{\mathrm{N} 1}+\mathrm{F}_{\mathrm{N} 2}=\mathrm{F}^{\star} \cos \alpha$

### 2.2. Formula for calculating the total gradient

For calculating the total (maximum) gradient of a weighing site the following formula may be used:

x is transversal to the driving direction $y$ is the driving direction $z$ is the elevation of the site surface
$\varphi$ is the angle between the $x$ and the $y$ axis for which the elevation $z$ shall be calculated.

The elevation $z$ is the sum of the partial elevations in $x$ and $y$ direction $\Delta z_{x}$ and $\Delta z_{y}$
$z=\Delta z_{x}+\Delta z_{y}$
$\Delta z_{\mathrm{x}}=\mathrm{z}_{\mathrm{x}}{ }^{*} \cos (\varphi)$
$\Delta z_{y}=z_{y}{ }^{*} \sin (\varphi)$
$(12) \cap(13) \cap(14):$
$z=z_{x}{ }^{*} \cos (\varphi)+z_{y}{ }^{*} \sin (\varphi)$
In order to find the maximum elevation (corresponding to the total gradient) the equation (15) in z must be derived in $\varphi$ and set to zero:
$\mathrm{dz} / \mathrm{d} \varphi=\mathrm{Z}_{\mathrm{x}}{ }^{*} \sin (\varphi)-\mathrm{Z}_{\mathrm{y}}{ }^{*} \cos (\varphi)=0$
The transformation of the formula (16) into an equation in $\varphi$ is not completed here because it might not be possible in a simple way. For some special cases the validity of the formulas (15) and(16) can easily be proven:

| $z_{x}$ | $z_{y}$ | $\varphi$ | $z$ |
| :--- | :--- | :--- | :--- |
| 0.1 | 0 | $0^{\circ}$ | 0.1 |
| 0 | 0.1 | $90^{\circ}$ | 0.1 |
| 0.1 | 0.1 | $45^{\circ}$ | 0.14 |

For other values for $z_{x}$ and $z_{y}$ it might be easiest to use equation (15) and trying out different values for $\varphi$ until the maximum for $z$ is found.

Instead of $z_{x}$ and $z_{y}$ the gradients cross and lengthwise in [\%] may also be input in order to get the total gradient $z$ directly in [\%] values.
3. Calculation of the Indication Difference caused by the Gradient of the Weighing Site

### 3.1 Measuring the Gross Weight

The difference to the indication on level ground is calculated according equation (11). The difference is very small and may be neglected in most cases. At a gradient of $5 \%$ it is only $-0.12 \%$ of the gross weight.

### 3.2 Measuring the Axle and the Wheel Weight

The height of the centre of gravity may affect the load on a individual wheel or axle. The shorter the vehicle, the narrower the track and the higher the position of the centre of gravity the bigger the effect.

If weighing vehicles with liquid payload also the position of the centre of gravity may change as well, causing additional differences. A precise prediction of the magnitude of this effect is difficult.

The following graph was calculated for quick determination of the influence of a gradient on the weight on wheels and axles. It is valid for vehicles with solid payload. The calculation was made for a $3 \%$ gradient. The graduations for $1 \%$ and $2 \%$ are approximate values with an error within the reading uncertainty. The use of the graph is explained by the following examples.

Weight differences in relation to the actual axle and wheel load at gradients of $1 \%, 2 \%$ and $3 \%$


The use of the graph is explained by the following examples. Example 1 and 2 refer to a gradient in driving direction (uphill, downhill).

## Examples

1.     -         - Determination of the difference of the axle weight of a vehicle with a wheel base of $I=6.5 \mathrm{~m}$ on a $2 \%$ uphill slope. The position of the centre of gravity is 2 m in front of the rear axle ( $b=2 \mathrm{~m}, a=I-b=4,5 \mathrm{~m}$ ) and 1.5 m over the road surface ( $h=1.5 \mathrm{~m}$ ).

This results in the following ratios:
$a / l=4,5 / 6,5=0,69 \quad h / l=1,5 / 6,5=0,23$
The graph shows the following differences:
front axle: $\quad$ Load change $=-1,5 \%$
rear axle: $\quad$ Load change $=+0,7 \%$
2. - - - Determination of the difference of the axle weight of a vehicle with a wheel base of $I=4 \mathrm{~m}$ on a $3 \%$ downhill slope. The position of the centre of gravity is 1 m in front of the rear axle ( $b=1$ $\mathrm{m}, a=I-b=3 \mathrm{~m})$ and 2 m over the road surface ( $h=2 \mathrm{~m}$ ).
Note: This is an abnormal case as $75 \%$ of the weight is concentrated on the rear axle! Also the lengthwise gradient of $3 \%$ is far beyond common practice!

This results in the following ratios:
$a / l=3 / 4=0,75 \quad h / l=2 / 4=0,5$
The graph shows the following differences:
front axle: Load change $=+6,0 \%$
rear axle: $\quad$ Load change $=-2,0 \%$

The graph may also be used for determining the differences of wheel loads if weighing on a site with a gradient crosswise.
Note: Usually the wheel weight is only measured for determining the axle load. In this case all differences are fully compensating each other!

## Example:

3.- -. -Determination of the difference of the wheel weight of a vehicle with a track width of $I=2 \mathrm{~m}$ on a $1 \%$ crosswise slope. The position of the centre of gravity is 0.1 m out of the centre line towards the upper wheels ( $b=1.1 \mathrm{~m}, a=I-b=0.9 \mathrm{~m}$ ) and 2 m over the road surface ( $h=2 \mathrm{~m}$ ).

This results in the following ratios:
$\mathrm{a} / \mathrm{I}=0,9 / 2,0=0,45 \quad \mathrm{~h} / \mathrm{I}=2 / 2=1$
The graph shows the following differences:
higher wheel: $\quad$ Load change $=-1,8 \%$
lower wheel: $\quad$ Load change $=+2,2 \%$

## 4. Conclusion

The gross vehicle weight indication is not affected as long as the maximum gradient stated in the user manual of the scale is respected in order to prevent from malfunction.

The same is true for the axle weight as long as the vehicle is of common dimensions, loaded in a way that the load is not extremely concentrated and weighed on a site which is reasonably horizontal in the driving direction. A site may be considered as horizontal if a vehicle does not roll away at once when the brakes are released.

The wheel weights are most susceptible for tilting of the weighing site in crosswise direction. The effect compensates fully if the wheel weights are used for calculating the axle weight only.

In the case of vehicles carrying liquid payload the same is true as long as the tank is full, empty or divided vertically into small sections. Additional differences may occur in the case that the tank is only partially filled due to the displacement of the centre of gravity. A precise prediction of the effect is difficult because it is depending on the form of the tank and of the fluid level.

Absolute worst case:
The following assumption was made in order to give an idea what differences are possible in the worst case. Such a vehicle may be for example a movable machine or crane not intended to circulate in traffic. Such a vehicle may have a wheel base of 4 m , an average track width of 1.6 m and its centre of gravity 1 m ahead of the rear axle, 0.2 m off the lengthwise centreline and 2.5 m above the road surface. A gradient of $2 \%$ will result in the following load differences:

Total weight: The effect is negligible
Axle load: The effect is negligible as long as the slope is crosswise to the driving direction. With a slope of $2 \%$ lengthwise the following differences in relation to the actual axle loads will occur: $+/-1,6 \%$ for the rear axle and $-/+5 \%$ for the front axle. (First sign: uphill; second sign: downhill.)

Wheel load: The biggest effect occurs in the case of a crosswise gradient. The difference in relation to the actual wheel load is $-/+2,8 \%$ for the more heavily loaded and $+/-3.6 \%$ for the less heavily loaded wheel. (First sign: centre of gravity shifted towards the higher wheel; second sign: towards the lower wheel).

The user may reduce all above effects to a big extent by selecting an appropriate weighing site.

